Statistical Analysis of Location Parameter of Inverse Gaussian Distribution Under Noninformative Priors

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Abstract

Bayesian estimation for location parameter of the inverse Gaussian distribution is presented in this paper. Noninformative priors (Uniform and Jeffreys) are assumed to be the prior distributions for the location parameter as the shape parameter of the distribution is considered to be known. Four loss functions: Squared error, Trigonometric, Squared logarithmic and Linex are used for estimation. Bayes risks are obtained to find the best Bayes estimator through simulation study and real life data.

Keywords: Bayesian estimation, noninformative prior, Jeffreys prior, loss function, Bayes estimator, Bayes risk, simulation study.

AMS Classification Code: 62F15

1. Introduction

A lot of work has been done on the estimation of the parameters of the inverse Gaussian distribution. There are many applications of the inverse Gaussian distribution other than mathematical statistics. It is used in engineering to make quantitative analysis and to describe various phenomena.

Lindley (1980) suggested the ratio of two integrals which was based on the asymptotic approximation. Sinha (1986) using the diffuse prior which was focused on the re-parameterized and derived the marginal posteriors and the highest–posterior-density of parameters based on the Bayesian inferences. Ismail and Auda (2006) said that the

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inverse Gaussian distribution is a monotonic function, it first increases then decreases. They used Gibbs sampler and from the Gibbs sampler they found the posterior estimates. Lemeshko, et al. (2010) have discussed different quality adjustment tests for the family of inverse Gaussian distributions.

Murphy (2007) explained the conjugate prior for the inverse Gaussian distribution. He used different conjugate priors for the inverse Gaussian distribution. Meintanis (2008) presented an article in which he has given goodness of fit test for the family of symmetric normal variance of inverse Gaussian distribution is constructed. Ma, Liu and Ahmed (2013) notes the properties of Bayes shrinkage estimator and its uses for the dispersion of inverse Gaussian model. He considered the random sample of size n which is drown from inverse Gaussian distribution and the unbiased estimates of its parameter have found. Aminzadah (2011) has used two methods of approximation for the renewal process of inverse Gaussian distribution renewal process.

Stogiannis and Croni (2012) present that the inverse Gaussian distribution is often used for modeling. But he used the tests for outliers' parameter of inverse Gaussian distribution in which the shape parameter μ following F-statistic distribution that turned into normal approximations for unequal samples. Pandey and Rao (2010) have given the Bayesian estimation of the parameter of inverse Gaussain distribution using Morkov chain Monte Carlo Methods. Feroze (2012) has given Bayesian analysis to the scale parameters of inverse Gaussian distribution. More details can be seen in Aminzadeh (2011), and Khan (2014).

Model and Likelihood Function

A random variable **x** is said to possess an inverse Gaussian distribution if its p.d.f has the following form

$$f(x)=\sqrt{\frac{\lambda}{2\pi x^{3}}}e^{-\lambda\frac{(x-\mu)^{2}}{2x\mu^{2}}}\,,0<x<\infty\,,0<\mu<\infty,0<\lambda<\infty \tag{1}$$

Let $x=x_1,x_2,...,x_n$ be a random sample taken from inverse Gaussian distribution with unknown location parameter μ and known shape parameter λ then the likelihood function is:

$$L(x;\mu)=(\frac{\lambda}{2\pi})^{\frac{n}{2}}\prod_{i=1}^{n}x_i^{-\frac{3}{2}}e^{-\frac{\lambda}{2\mu^{2}}\sum_{i=1}^{n}\frac{(x_i-\mu)^{2}}{x_i}} \tag{2}$$

2. Bayes Estimators and Bayes Risks under Different Loss Functions

This section contains Bayes estimators (BEs) and Bayes risks (BRs) under different loss functions. Four loss functions are used which are defined below

The Squared Error Loss Function (SELF)

The SELF is defined as:

$$L_1 = L_1(\mu, \mu^*) = (\mu - \mu^*)^2$$

where μ^* is the Bayes estimator of parameter μ .

Now the BE is obtained after minimizing the expectation (with respect to posterior distribution) of this loss function.

$$\mu^* = E(\mu \mid x)$$

And the Bayes risk is obtained as

$$\rho(\mu^*) = E_{x|\mu}\left\{L_1(\mu, \mu^*)\right\} = Var(\mu \mid x)$$

The Trignometric loss function is given as:

$$L_2 = L_2(\mu, \mu^*) = \cosh\left\{q(\mu - \mu^*)\right\} - 1, q \neq 0$$

where μ^* is the estimator of parameter μ .

Now the BE is obtained after minimizing the expectation (with respect to posterior distribution) of this loss function.

$$\mu^* = \frac{1}{2q} Log\left[\frac{E_{\mu|x}(e^{qu})}{E_{\mu|x}(e^{-qu})}\right]$$

And the Bayes posterior risk is obtained as:

$$\rho(\mu^*) = \sqrt{E_{\mu|x}(e^{q\mu})E_{\mu|x}(e^{-q\mu})} - 1$$

The Sequared Logarithmic loss function is given as:

$$L_3 = L_3(\mu, \mu^*) = (\log \mu^* - \log \mu)^2$$

where μ^* is the estimator of parameter μ .

Now the BE is obtained after minimizing the expectation (with respect to posterior distribution) of this loss

$\mu^* = e^{E_{\mu|x}(\log \mu)}$

And, the Bayes risk is obtained as

$\rho(\mu^*) = E_{\mu|x}(\log \mu)^2 - \left\{E_{\mu|x}(\log \mu)^2\right\}$

The Linex loss function is defined as :

$L_4 = L_4(\mu, \mu^*) = e^{t(\mu^* - \mu)} - t(\mu^* - \mu) - 1, t \neq 0$

where μ^* is the estimator of parameter μ .

Now the BE is obtained after minimizing the expectation (with respect to posterior distribution) of this loss function.

$\mu^* = -\frac{1}{t} Log\left[E_{\mu|x}(e^{-t\mu})\right] + tE_{\mu|x}(\mu)$

And the Bayes posterior risk is obtained as:

$\rho(\mu^*) = Log\left[E_{\mu|x}(e^{-t\mu})\right] + tE_{\mu|x}(\mu)$

3. Bayesian Analysis Under Uniform Prior:

The non-informative Uniform prior (UP) of parameter μ is defined as:

$p(\mu) \propto 1, 0 < \mu < \infty$ (3)

The posterior distribution of parameter μ for the given data:

$x = x_1, x_2, ..., x_n$ using (2) and (3) is

$$p(\mu | x) = \frac{1}{k_1} e^{-\frac{\lambda}{2\mu^2}\sum_{i=1}^{n}\frac{(x_i - \mu)^2}{x_i}}, 0 < \mu < \infty$$

where $k_1 = \int_0^\infty e^{-\frac{\lambda}{2\mu^2}\sum_{i=1}^{n}\frac{(x_i - \mu)^2}{x_i}} d\mu.$ (4)

which is not in closed form, so we solve it numerically.

Expressions for BEs and BRs under Different LFs

$$\mu^*_1 = \frac{1}{k_1}\int_0^\infty \mu e^{-\frac{\lambda}{2\mu^2}\sum_{i=1}^{n}\frac{(x_i - \mu)^2}{x_i}} d\mu.$$

$$\rho(\mu_1^*) = \frac{1}{k_1}\int_0^\infty \mu^2 e^{-\frac{\lambda}{2\mu^2}\sum_{i=1}^{n}\frac{(x_i - \mu)^2}{x_i}} d\mu - (\mu^*_1)^2$$

$$\mu_2^* = \frac{1}{2q} Log\left(\frac{\int_0^\infty e^{q\mu - \frac{\lambda}{2\mu^2}A} d\mu}{\int_0^\infty e^{-q\mu - \frac{\lambda}{2\mu^2}A} d\mu}\right)$$

$$\rho(\mu_2^*) = \frac{1}{k_1}\sqrt{\int_0^\infty e^{q\mu - \frac{\lambda}{2\mu^2}A} d\mu \int_0^\infty e^{-q\mu - \frac{\lambda}{2\mu^2}A} d\mu} - 1$$

where $A = \sum_{i=1}^{n} \frac{(x_i - \mu)^2}{x_i}$

$$\mu_3^* = e^{\frac{1}{k_1}\int_0^\infty Log(\mu) e^{-\frac{\lambda}{2\mu^2}\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{x_i}} d\mu}$$

$$\rho(\mu_3^*) = \frac{1}{k_1}\int_0^\infty Log(\mu)^2 e^{-\frac{\lambda}{2\mu^2}\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{x_i}} d\mu - (A_1)^2$$

where $A_1 = \frac{1}{k_1}\int_0^\infty Log(\mu) e^{-\frac{\lambda}{2\mu^2}\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{x_i}} d\mu$

$$\mu_4^* = -\frac{1}{t} Log\left(\int_0^\infty e^{t\mu - \frac{\lambda}{2\mu^2}A} d\mu\right)$$

$$\rho(\mu_4^*) = Log\left(\int_0^\infty e^{t\mu - \frac{\lambda}{2\mu^2}A} d\mu\right) + tX_1$$

where $A = \sum_{i=1}^{n} \frac{(x_i - \mu)^2}{x_i}, X_1 = \frac{1}{k_1}\int_0^\infty \mu e^{-\frac{\lambda}{2\mu^2}\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{x_i}} d\mu$

4. Bayesian Analysis under Jeffreys Prior

The Jeffreys prior (JP) for the parameter μ is given below:

$$p(\mu) \propto \mu^{-\frac{3}{2}}, 0 < \mu < \infty \tag{5}$$

The posterior distribution of μ for the given data set $x = x_1, x_2, ..., x_n$ using (2) and (5) is:

$$p(\mu \mid x) = \frac{1}{k_2}\mu^{-\frac{3}{2}} e^{-\frac{\lambda}{2\mu^2}\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{x_i}}, 0 < \mu < \infty$$

where $k_2 = \int_0^\infty \mu^{-\frac{3}{2}} e^{-\frac{\lambda}{2\mu^2}\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{x_i}} d\mu$ (6)

Expressions For BEs and PRs Under Different LFs

$$\mu_1^* = \frac{1}{k_2}\int_0^\infty \mu^{-\frac{1}{2}} e^{-\frac{\lambda}{2\mu^2}\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{x_i}} d\mu,$$

$$\rho(\mu_1^*) = \frac{1}{k_2}\int_0^\infty \mu^{-\frac{1}{2}} e^{-\frac{\lambda}{2\mu^2}\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{x_i}} d\mu - (\widehat{\mu}_1^*)^2$$

$$\mu_2^* = \frac{1}{2q} Log\left(\frac{\int_0^\infty \mu^{-\frac{3}{2}} e^{q\mu-\frac{\lambda}{2\mu^2}A} d\mu}{\int_0^\infty \mu^{-\frac{3}{2}} e^{-q\mu-\frac{\lambda}{2\mu^2}A} d\mu}\right)$$

$$\rho(\mu_2^*) = \frac{1}{k_2}\sqrt{\int_0^\infty \mu^{-\frac{3}{2}} e^{q\mu-\frac{\lambda}{2\mu^2}A} d\mu \int_0^\infty \mu^{-\frac{3}{2}} e^{-q\mu-\frac{\lambda}{2\mu^2}A} d\mu} - 1$$

where $A = \sum_{i=1}^{n}\frac{(x_i-\mu)^2}{x_i}$

$$\mu_3^* = e^{\frac{1}{k_2}\int_0^\infty Log(\mu)\mu^{-\frac{3}{2}} e^{-\frac{\lambda}{2\mu^2}\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{x_i}} d\mu}$$

$$\rho(\mu_3^*) = \frac{1}{k_2} Log(\mu)^2 \mu^{-\frac{3}{2}} e^{-\frac{\lambda}{2\mu^2}\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{x_i}} d\mu - (A_2)^2$$

Where $A_2 = \frac{1}{k_2}\int_0^\infty Log(\mu)\mu^{-\frac{3}{2}} e^{-\frac{\lambda}{2\mu^2}\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{x_i}} d\mu$

$$\mu_4^* = -\frac{1}{t} Log\left(\int_0^\infty \mu^{-\frac{3}{2}} e^{t\mu-\frac{\lambda}{2\mu^2}A} d\mu\right)$$

$$\rho(\mu_4^*) = Log\left(\int_0^\infty \mu^{-\frac{3}{2}} e^{t\mu-\frac{\lambda}{2\mu^2}A} d\mu\right) + tX_2$$

where $A = \sum_{i=1}^{n}\frac{(x_i-\mu)^2}{x_i}, X_2 = \frac{1}{k_2}\int_0^\infty \mu^{-\frac{1}{2}} e^{-\frac{\lambda}{2\mu^2}\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{x_i}}$

Posterior Predictive Distribution

The posterior predictive distribution is derived using informative and noninformative priors. Let $Y = X_{n+1}$ be the future random variable given the sample observation $x = x_1, x_2, ..., x_n$ from inverse Gaussian distribution with unknown parameter μ. Posterior predictive distribution under noninformative priors also has no closed form so we also solve it numerically.

$$p(y \mid \underline{x}) = \sqrt{\frac{\lambda}{2\pi y^3}} e^{-\lambda \frac{(y-\mu)^2}{2y\mu^2}}, 0 < y < \infty, 0 < \mu < \infty, 0 < \lambda < \infty$$

The posterior predictive under non informative prior is:

$$p_U(y \mid \underline{x}) = \frac{1}{k_1} \int_0^\infty \sqrt{\frac{\lambda}{2\pi y^3}} e^{-\frac{\lambda}{2\mu^2}\left[\frac{(y-\mu)^2}{y} + \sum_{i=1}^{n} \frac{(y_i-\mu)^2}{y_i}\right]} d\mu, 0 < y < \infty$$

where k_1 is defined in (4)

$$p_J(y \mid \underline{x}) = \frac{1}{k_2} \int_0^\infty \sqrt{\frac{\lambda}{2\pi y^3}} \mu^{-\frac{3}{2}} e^{-\frac{\lambda}{2\mu^2}\left[\frac{(y-\mu)^2}{y} + \sum_{i=1}^{n} \frac{(y_i-\mu)^2}{y_i}\right]} d\mu, 0 < y < \infty$$

where k_2 is defined in (6).

5. Simulation Study

Simulation has many properties that whether the data are discrete or continuous. When the analytic solution of the problems may be difficult or impossible, simulation can provide an effective way to handle. So by taking different values of parameter μ and fixing the values of $\lambda = 3, 5, 8$ along different sample sizes BE s and BRs are obtained. This section presents simulation study of Bayes estimators and Bayes risks. It is clear from the above results that by increasing sample size the Bayes estimator approaches to its true value of parameter. For small sample size, the estimators are underestimated and thus by increasing sample size more accuracy and precision obtained due to decreasing the BRs. By increasing the value of the shape parameter, risk decreases for all loss functions. The LINEX loss function (LLF) is recommended for further use of estimation as it has minimum risk.

By comparing the priors which we have used, it is clear that the JP gives the smallest risk for all loss functions.

Table 1: Simulation of BEs and BRs under different LFs when $\mu = 3, 4,$ and $\lambda = 3$

μ	NIP	n	SELF	TLF	SLLF	LLF
3	UP	30	3.46233 (2.06163)	3.44725 (0.000010)	3.43796 (1.94637)	3.46879 (0.00010)
		100	3.09582 (0.11345)	3.10192 (5.71227×10^{-6})	3.06488 (0.01091)	3.10390 (5.71780×10^{-6})
		300	3.03110 (0.03226)	3.02449 (1.60194×10^{-6})	3.03162 (0.00344)	3.03413 (1.61736×10^{-6})
		500	3.01612 (0.018720)	3.02871 (9.48622×10^{-7})	3.01051 (0.00203)	3.01833 (9.45934×10^{-7})
	JP	30	3.14578 (0.03985)	3.15628 (0.00003)	3.10632 (0.03928)	3.17986 (0.00003)
		100	3.04608 (0.10382)	3.04092 (5.17213×10^{-6})	3.03951 (0.01046)	3.04531 (5.18652×10^{-6})
		300	3.01461 (0.03141)	3.01008 (1.56228×10^{-6})	2.99789 (0.00337)	3.01814 (1.57514×10^{-6})
		500	3.01002 (0.01855)	3.01199 (9.3088×10^{-7})	3.00413 (0.00202)	3.00730 (9.19587×10^{-7})
4	UP	30	4.6178 (1.45627)	5.24198 (0.00090)	4.74922 (0.10406)	5.09712 (0.00073)
		100	4.17039 (0.29164)	4.19766 (0.00002)	4.18780 (0.01531)	4.15638 (0.00001)
		300	4.07015 (0.07928)	4.04009 (3.8732×10^{-6})	4.05392 (0.00463)	4.05456 (3.9204×10^{-6})

		500	4.03161 (0.04518)	4.03628 (2.2651×10^{-6})	4.02181 (0.00273)	4.03175 (2.25747×10^{-6})
	JP	30	4.24577 (0.05742)	4.36648 (0.00011)	4.19608 (0.056651)	4.38381 (0.00011)
		100	4.08088 (0.25886)	4.06430 (0.00001)	4.06234 (0.014142)	4.09027 (0.00001)
		300	4.04882 (0.07687)	4.03263 (3.79627×10^{-6})	4.03866 (0.00455)	0.07687 (3.77039×10^{-6})
		500	4.02845 (0.0446)	4.02059 (2.22032×10^{-6})	4.00205 (0.00269)	4.00965 (2.20275×10^{-6})

Note: Where in brackets show the posterior risks.

Table 2: Simulation of BEs and BRs under different LFs when $\mu = 3, 4,$ and $\lambda = 5$

μ	**NIP**	**n**	**SELF**	**TLF**	**SLLF**	**LLF**
	UP	30	3.19395 (0.30360)	3.19429 (0.00002)	3.16040 (0.02448)	3.20888 (0.00002)
		100	3.05780 (0.018722)	3.05486 (3.07617×10^{-6})	3.04543 (0.00204)	3.05537 (3.08186×10^{-6})
		300	3.01391 (0.018722)	3.02176 (9.42951×10^{-7})	3.01349 (0.00204)	3.01440 (9.36348×10^{-7})
		500	3.01052 (0.01081)	3.00513 (5.52714×10^{-7})	3.01013 (0.00123)	3.00699 (5.45756×10^{-7})
		30	3.04407	3.12465	3.07667	3.11659

	JP		(0.02172)	(0.000012)	(0.02197)	(0.00001)
		100	3.02435 (0.05855)	3.025044 (2.93235×10^{-6})	3.01795 (0.006150)	3.042641 (2.98109×10^{-6})
		300	3.00964 (0.01853)	3.00467 (9.16396×10^{-7})	3.00542 (0.00202)	3.01278 (9.28785×10^{-7})
		500	3.00649 (0.01079)	3.01239 (5.40861×10^{-7})	3.00412 (0.00121)	3.01033 (5.60799×10^{-7})
		30	4.47257 (1.15605)	4.42779 (0.00005)	4.30247 (0.03604)	4.41757 (0.00005)
		100	3.05780 (0.06178)	3.05486 (3.07617×10^{-6})	3.04543 (0.00633)	3.05537 (3.08186×10^{-6})
	UP	300	4.02309 (0.04485)	4.03157 (2.25701×10^{-6})	4.03374 (0.00273)	4.02931 (2.25284×10^{-6})
		500	4.0243 (0.02651)	4.00985 $(1.314945 \times 10^{-6})$	4.01597 (0.00162)	4.00934 (1.31385×10^{-6})
4		30	4.11856 (0.03023)	4.17439 (0.000033)	4.131882 (0.03035)	4.15762 (0.00003)
		100	4.05513 (0.14395)	4.04832 (7.1606×10^{-6})	4.03639 (0.00828)	4.041043 (7.1272×10^{-6})
	JP	300	4.01028 (0.04406)	4.02245 (2.22349×10^{-6})	4.00736 (0.00269)	4.00629 (2.19624×10^{-6})
		500	4.02157 (0.02641)	4.00870	4.01019 (0.00161)	4.00834

				(1.30817×10^{-6})		(1.3077×10^{-6})

Table 3: Simulation of BEs and BRs under different LFs when $\mu = 3, 4,$ and $\lambda = 8$

μ	NIP	N	SELF	TLF	SLLF	LLF
3	UP	30	3.13357 (0.15321)	3.12455 (7.55843×10^{-6})	3.10073 (0.01407)	3.13601 (7.66761×10^{-6})
		100	3.03721 (0.03670)	3.03915 (1.83841×10^{-6})	3.03058 (0.00388)	3.03043 (1.82488×10^{-6})
		300	3.01111 (0.01141)	3.01320 (5.84387×10^{-7})	3.00747 (0.00128)	3.01135 (5.79837×10^{-7})
		500	3.01399 (0.00605)	3.01415 (3.32898×10^{-7})	3.01369 (0.00072)	3.00854 (3.29584×10^{-7})
	JP	30	3.06623 (0.01330)	3.06681 (6.81449×10^{-6})	3.03789 (0.01318)	3.065810 (6.81150×10^{-6})
		100	3.01380 (0.03542)	3.01916 (1.78132×10^{-6})	3.00462 (0.00380)	3.009811 (1.76352×10^{-6})
		300	3.00572 (0.01137)	2.99930 (5.78468×10^{-7})	3.00310 (0.00126)	3.00907 (5.83986×10^{-7})
		500	3.00458 (0.00602)	3.01026 (3.23845×10^{-7})	3.00917 (0.00072)	3.01548 (3.10867×10^{-7})
		30	4.25162 (0.41538)	4.25322 (0.00002)	4.18278 (0.01964)	4.22129 (0.00002)

4	UP	100	4.06656 (0.08956	4.08035 $\left(4.52488 \times 10^{-6}\right)$	4.03730 (0.00521)	4.04743 $\left(4.41206 \times 10^{-6}\right)$
		300	4.02426 (0.02772)	4.02276 $\left(1.38308 \times 10^{-6}\right)$	4.01679 (0.00169)	4.02715 $\left(1.38929 \times 10^{-6}\right)$
		500	4.00862 (0.01634)	4.01192 $\left(8.19953 \times 10^{-7}\right)$	4.01792 (0.00101)	4.01197 $\left(8.16881 \times 10^{-7}\right)$
	JP	30	4.06285 (0.01789)	4.10774 (0.00002)	4.05025 (0.01784)	4.120991 (0.00002)
		100	4.02577 (0.08545)	4.03289 $\left(4.29642 \times 10^{-6}\right)$	4.01350 (0.00509)	4.02418 $\left(4.2672 \times 10^{-6}\right)$
		300	3.99575 (0.02610)	4.00920 $\left(1.36274 \times 10^{-6}\right)$	4.00747 (0.00168)	4.01798 $\left(1.37235\times 10^{-6}\right)$
		500	4.01391 (0.01631)	4.00121 $\left(8.06336 \times 10^{-7}\right)$	4.00203 (0.00101)	4.00874 $\left(8.13513 \times 10^{-7}\right)$

6. Bayes Estimates and Bayes Risks for Data

Chhikara and Folks (1974) analyzed the maintenance data which represents active repair times (in hours) for an airborne communication transceiver, 46 observations are given as follows:

Table 4: Repair Times (in hours) of 46 Transceivers

0.2	0.5	0.5	0.6	0.7	0.3	0.5	0.5	0.6	0.7
1.3	0.8	1.0	1.0	1.1	1.5	0.7	0.8	1.0	1.0
1.5	1.5	1.5	2.0	2.0	2.2	2.5	2.7	3.0	3.0
3.3	3.3	4.0	4.0	4.5	4.7	5.0	5.4	5.4	7.0
7.5	8.8	9.0	10.3	22.0	24.5				

Table 5: BEs and BRs under different LFs using Real Data Set

Priors		SELF	TLF	SLLF	LLF
UP	μ^*	3.96173	3.96175	3.95851	3.89052
	$\rho(\mu^*)$	0.64813	0.00003	0.03468	0.00003
JP	μ^*	3.76580	3.76581	3.70893	3.76344
	$\rho(\mu^*)$	0.47233	0.00002	0.02942	0.00002

The results obtained from real data set conform the results of simulation study. Thus it is clear that Jeffreys prior is suitable prior for location parameter as BE has minimum risk and the BE under LLF is the best estimator as it has minimum risk.

7. Conclusions

In this study, loss functions: Squared, Trignometric, Squared Logarithmic, Linex are used for the estimation of location parameter of the inverse Gaussian distribution. Non-informative priors are assumed for the location parameter. Linex loss function is recommended for estimation of location parameter as it has minimum risk. We observed that risk depends on sample size, as sample size increases, risk decreases. Further, the Jeffreys prior is suitable prior for location parameter as BE has minimum risk.

References

Aminzadeh, M. S. (2011). Bayesian estimation of renewal function for inverse Gaussian renewal process. *Journal of Statistical Computation and Simulation*, *81*(3), 331-341. https://doi.org/10.1080/00949650903325153.

Chhikara, R. S., & Folks, J. L. (1974). Estimation of the inverse Gaussian distribution function, *Journal of the American Statistical Association*, 69(345), 250-254.

Feroze, N. (2012). Estimation of scale parameter of inverse Gausian distribution under a Bayesian framework using different loss functions. *Scientific Journal of Review, 1*(3), 39-52.

Ismail, S. A., & Auda, H. A. (2006). Bayesian and fiducial inference for the inverse Gaussian distribution via Gibbs sampler. *Journal of Applied Statistics*, *33*(8), 787-805. https://doi.org/10.1080/02664760600742268.

Khan, N. (2014), *Statistical Analysis of Inverse Gaussian Distribution in Bayesian Framework* (Unpublished MPhil Thesis), Quaid-i-Azam University, Islamabad, Pakistan.

Lemeshko, B. Y., Lemeshko, S. B., Akushkina, K. A., Nikulin, M. S., Saaidia, N., Saaidia N. (2010). Inverse Gaussian model and its applications in reliability and survival analysis. In *Mathematical and statistical models and methods in reliability* (pp. 433-453). Birkhäuser, Boston, MA: United States. https://doi.org/10.1007/978-0-8176-4971-5 33,

Lindley, D. V. (1980). Approximate Bayesian methods. *Trabajos De Estadística Y De Investigación Operativa*, *31*(1), 223-245. https://doi.org/10.1007/BF02888353.

Ma, T., Liu, S., & Ahmed, S. E. (2014). Shrinkage estimation for the mean of the inverse Gaussian population. *Metrika*, *77*(6), 733-752. https://doi.org/10.1007/s00184-013-0462-8.

Meintanis, S. G. (2008). Tests of fit for normal variance inverse Gaussian distributions. In *Proceedings of the World Congress on Engineering* (Vol. 2) London, U.K.

Pandey, H., & Rao, A. K. (2010). Bayesian estimation of scale parameter of inverse Gaussian distribution using linex loss function. *Journal of Computer and Mathematical Sciences*, *1*(2), 103-273.

Stogiannis, D., & Caroni, C. (2012). Tests for outliers in the inverse Gaussian distribution, with application to first hitting time models. *Journal of Statistical Computation and Simulation*, *82*(1), 73-80. https://doi.org/10.1080/00949655.2010.527843.

Sinha, S. K. (1986). Bayesian estimation of the reliability function of the Inverse Gaussian distribution. *Statistics & Probability Letters*, *4*(6), 319-323. https://doi.org/10.1016/0167-7152(86)90052-0.

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